

Mark Scheme (Results)

Summer 2013

GCE Further Pure Mathematics 2 (6668/01R)





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- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

- 1. The total number of marks for the paper is 75.
- 2. The Edexcel Mathematics mark schemes use the following types of marks:
- M marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.
- 3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod benefit of doubt
- ft follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao correct answer only
- cso correct solution only. There must be no errors in this part of the question to obtain this mark
- isw ignore subsequent working
- awrt answers which round to
- SC: special case
- oe or equivalent (and appropriate)
- dep dependent
- indep independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- The second mark is dependent on gaining the first mark
- 4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
- 5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
- 6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
- 7. Ignore wrong working or incorrect statements following a correct answer.
- 8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme

General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles). Method mark for solving 3 term quadratic:

1. Factorisation

$$(x^2 + bx + c) = (x + p)(x + q)$$
, where $|pq| = |c|$, leading to x =

$$(ax^2 + bx + c) = (mx + p)(nx + q)$$
, where $|pq| = |c|$ and $|mn| = |a|$, leading to x =

2. <u>Formula</u>

Attempt to use <u>correct</u> formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving
$$x^2 + bx + c = 0$$
: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c$, $q \neq 0$, leading to $x = ...$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n \rightarrow x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n \rightarrow x^{n+1})$

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an <u>exact</u> answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these \underline{may} not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Question Number	Scheme	Marks
1.	$z = x \qquad w = \frac{x + 2i}{ix}$	M1A1
	$w = \frac{1}{i} + \frac{2i}{ix}$	
	$u + iv = -i + \frac{2}{x}$	
	$\left(u = \frac{2}{x}\right) \qquad v = -1$	M1
	$\therefore w$ is on the line $v+1=0$	A1
		4 Marks

NOTES

M1 for replacing at least one z with x to obtain (ie show an appreciation that y = 0)

A1
$$w = \frac{x+2i}{ix}$$

M1 for writing w as u + iv and equating real or imaginary parts to obtain either u or v in terms of x or just a number

A1 for giving the equation of the line v+1=0 oe must be in terms of v

Question Number	Scheme	Marks
1 (0111.001	Q1 - ALTERNATIVE 1:	
	$w = \frac{x + iy + 2i}{i(x + iy)}$ Replacing z with x+iy	
	$w = \frac{x + iy + 2i}{-y + ix} \times \frac{-y - ix}{-y - ix}$	
	$w = \frac{(x + i(y + 2))(-y - ix)}{y^2 + x^2}$	
	$w = \frac{2x - i(x^2 + y^2 + 2y)}{y^2 + x^2}$	
	$w = \frac{2x - ix^2}{x^2} = \frac{2}{x} - i$ Using $y = 0$. This is where the first M1 may be	M1A1
	awarded. A1 if correct even if expression is unsimplified but denominator must be real	
	v = -1 M1, A1 as in main scheme	M1A1
	Q1 - ALTERNATIVE 2:	
	$z = \frac{2i}{iw-1}$ Writing the transformation as a function of w	
	$z = \frac{2i}{i(u+iv)-1}$	
	$z = \frac{2i}{(-v-1)+iu} \times \frac{(-v-1)-iu}{(-v-1)-iu}$	
	$z = \frac{2u + 2i(-v - 1)}{(-v - 1)^2 + u^2} = \frac{2u}{(-v - 1)^2 + u^2} + i\left(\frac{2(-v - 1)}{(-v - 1)^2 + u^2}\right)$	
	$\left(\frac{2(-v-1)}{(v-1)^2 + u^2}\right) = 0 \text{ or simply } -2(v+1) = 0 \qquad \text{Using } y = 0 \text{ . This is}$ where the first M1 may be awarded. A1 if correct even if expression is	M1A1
	unsimplified but denominator must be real	
	v = -1 M1, A1 as in main mark scheme above	M1A1

Questi Numb	ion Der	Scheme	Marks
2.		NB Allow the first 5 marks with = instead of inequality	
		$\frac{6x}{3-x} > \frac{1}{x+1}$	
		$6x(3-x)(x+1)^{2} - (3-x)^{2}(x+1) > 0$	M1
		$(3-x)(x+1)(6x^2+6x-3+x) > 0$	
		(3-x)(x+1)(3x-1)(2x+3) > 0	M1dep
		Critical values $3, -1$	B1
		and $-\frac{3}{2}, \frac{1}{3}$	A1, A1
		Use critical values to obtain both of $-\frac{3}{2} < x < -1$ $\frac{1}{3} < x < 3$	M1A1cso
			7 Marks
NOTES	S		
M1	for n	nultiplying through by $(x+1)^2 (3-x)^2$	
	OR: in m	for collecting one side of the inequality and attempting to form a single fraction ark scheme)	(see alternative
 M1dep for collecting on one side of the inequality and factorising the result of the above (usual rules for factorising the quadratic) OR: for factorising the numerator - must be a three term quadratic - usual rules for factorising a quadratic (see alternative in mark scheme) Dependent on the first M mark 			
B1	for tl	he critical values 3, -1	
A1	for e	ither $-\frac{3}{2}$ or $\frac{1}{3}$	
A1 NB :	for the carrier for the carrie	he second of these ritical values need not be shown explicitly - they may be shown on a sketch or ju es or in the working for the ranges.	ist appear in the
M1	using must	g their 4 critical values to obtain appropriate ranges e.g. use a sketch graph of a x be the correct shape and cross the <i>x</i> -axis at the cvs) or a table or number line	quartic, (which
Alcso	fo	or both of $-\frac{3}{2} < x < -1$, $\frac{1}{3} < x < 3$	

Notes for Question 2 Continued

 $\left(-\frac{3}{2},-1\right)\cup\left(\frac{1}{3},3\right)$ All brackets must be round; if square brackets appear Set notation acceptable i.e.

anywhere then A0.

If both ranges correct, no working is needed for the last 2 marks, but any working shown must be correct.

Purely graphical methods are unacceptable as the question specifies "Use algebra...".

Q2 - ALTERNATIVE 1: $\frac{6x}{3-x} - \frac{1}{x+1} > 0$	
$\frac{6x(x+1) - (3-x)}{(3-x)(x+1)} > 0$	M1
$\frac{(3x-1)(2x+3)}{(3-x)(x+1)} > 0$	M1dep
Critical values 3, -1	B1
and $-\frac{3}{2}, \frac{1}{3}$	A1A1
Use critical values to obtain both of $-\frac{3}{2} < x < -1$ $\frac{1}{3} < x < 3$	M1A1cso
	7 Mar

Question Number	Scheme	Marks	
3(a)	$\frac{2}{(r+1)(r+3)} = \frac{A}{r+1} + \frac{B}{r+3}$		
	$2 = A(r+3) + B(r+1)$ $\frac{2}{(r+1)(r+3)} = \frac{1}{r+1} - \frac{1}{r+3}$	M1A1	
	N.B. for M mark you may see no working. Some will just use the "cover up" method to write the answer directly. This is acceptable.		(2)
(b)	$\sum \frac{2}{(r+1)(r+3)} = \sum \frac{1}{r+1} - \frac{1}{r+3}$		
	$= \left(\frac{1}{2} - \frac{1}{4}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) + \left(\frac{1}{4} - \frac{1}{6}\right) + \dots$		
	$+\left(\frac{1}{n-1} - \frac{1}{n+1}\right) + \left(\frac{1}{n} - \frac{1}{n+2}\right) + \left(\frac{1}{n+1} - \frac{1}{n+3}\right)$	M1A1ft	
	$=\frac{1}{2} + \frac{1}{3} - \frac{1}{n+2} - \frac{1}{n+3}$		
	$=\frac{5(n+2)(n+3)-6(n+3)-6(n+2)}{6(n+2)(n+3)}$	M1	
	$=\frac{5n^2+25n+30-12n-30}{6(n+2)(n+3)}$		
	$=\frac{n(5n+13)}{6(n+2)(n+3)} *$	A1	(4)
(c)	$\sum_{10}^{100} = \sum_{1}^{100} - \sum_{1}^{9}$	M1	
	$=\frac{100(500+13)}{6\times102\times103} - \frac{9\times58}{6\times11\times12} = \frac{1425}{1751} - \frac{29}{44} = 0.81382 0.65909$		
	= 0.1547 = 0.155	A1	(2)
		8 Marks	(2)

Notes for Ouestion 3 **Question 3a** M1 for attempting the PFs - any valid method for correct PFs $\frac{2}{(r+1)(r+3)} = \frac{1}{r+1} - \frac{1}{r+3}$ A1 N.B. for M mark you may see no working. Some will just use the "cover up" method to write the answer directly. This is acceptable. Award M1A1 if correct, M0A0 otherwise. **Question 3b** If all work in *r* instead of *n*, penalise last A mark only. M1 for using their PFs to list at least 3 terms at the start and 2 terms at the end so the cancelling can be seen. Must start at r = 1 and end at r = nA1ft for correct terms follow through their PFs M1 for picking out the (4) remaining terms and attempting to form a single fraction (unsimplified numerator with at least 2 terms correct) A1cso for $\frac{n(5n+13)}{6(n+2)(n+3)}$ * (Check all steps in the working are correct - in particular 3rd line from end in the mark scheme.) **NB:** If final answer reached correctly from $\frac{1}{2} + \frac{1}{3} - \frac{1}{n+2} - \frac{1}{n+3}$ (i.e. working shown from this point onwards) give 4/4 (even without individual terms listed)



Question Number	Scheme	Marks
4 (a)	$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 5y = 0$	
	$\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + y\frac{\mathrm{d}^3 y}{\mathrm{d}x^3} + 2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + 5\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	M1M1
	$\frac{\mathrm{d}^3 y}{\mathrm{d}x^3} = \frac{-5\frac{\mathrm{d}y}{\mathrm{d}x} - 3\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}}{y}$	A2,1,0 (4)
	Q4a – ALTERNATIVE 1:	
	$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \frac{-5y - \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}{y} = -5 - \frac{1}{y} \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$	
	$d^3y = 1 (dy)^3 = 2 (dy) (d^2y)$	M1M1
	$\frac{1}{\mathrm{d}x^3} = \frac{1}{y^2} \left(\frac{1}{\mathrm{d}x} \right) - \frac{1}{y} \left(\frac{1}{\mathrm{d}x} \right) \left(\frac{1}{\mathrm{d}x^2} \right)$	A2,1,0
(b)	When $x=0$ $\frac{dy}{dx}=2$ and $y=2$	
	$\frac{d^2 y}{dx^2} = \frac{1}{2} \left(-10 - 4 \right) = -7$	M1A1
	$\frac{d^3 y}{dx^3} = \frac{-10 - 3 \times 2 \times -7}{2} = 16$	A1
	$y = 2 + 2x - \frac{7}{2(!)}x^2 + \frac{16}{3!}x^3 + \dots$	M1
	2 2 7 2 8 3	A1
	$y = 2 + 2x - \frac{1}{2}x^2 + \frac{1}{3}x^3$	(5)
		9 Marks

Question Number	Scheme	Marks
	Alternative: $y = 2 + 2x + ax^2 + bx^3$	M1
	$(2+2x+ax^{2}+bx^{3})(2a+6bx)+(2+2ax+3bx^{2})^{2}$ +5(2+2x+ax^{2}+bx^{3})=0	M1
	Coeffs x^0 : $4a + 4 + 10 = 0$ $a = -\frac{7}{2}$	A1
	Coeffs x: $4a+12b+8a+10=0 \implies b=\frac{8}{3}$	A1
	$y = 2 + 2x - \frac{7}{2}x^2 + \frac{8}{3}x^3$	A1
NOTES		

Accept the dash notation in this question

Question 4a

M1 for using the product rule to differentiate
$$y \frac{d^2 y}{dx^2}$$

M1 for differentiating 5y and using the product rule or chain rule to differentiate $\left(\frac{dy}{dx}\right)^2$

A2,1,0 for
$$\frac{d^3 y}{dx^3} = \frac{-5\frac{dy}{dx} - 3\left(\frac{dy}{dx}\right)\frac{d^2 y}{dx^2}}{y}$$
 Give A1A1 if fully correct, A1A0 if **one** error and A0A0 if

more than one error. If there are two sign errors and no other error then give A1A0.

Do NOT deduct if the two
$$\frac{d^2y}{dx^2}$$
 terms are shown separately.

Alternative to Q4a

Can be re-arranged first and then differentiated.

M1M1 for differentiating, product and chain rule both needed (or quotient rule as an alternative to product rule)

A2,1,0 for
$$\frac{d^3 y}{dx^3} = \frac{1}{y^2} \left(\frac{dy}{dx}\right)^3 - \frac{2}{y} \left(\frac{dy}{dx}\right) \left(\frac{d^2 y}{dx^2}\right)$$

Give A1A1 if fully correct, A1A0 if **one** error and

A0A0 if more than one error

Question 4b for substituting $\frac{dy}{dx} = 2$ and y = 2 in **the equation** to obtain a numerical value for $\frac{d^2y}{dx^2}$ M1 for $\frac{d^2 y}{dr^2} = -7$ A1 for obtaining the correct value, 16, for $\frac{d^3y}{dr^3}$ A1 for using the series $y = f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \dots$ (2! or 2, 3! or 6) (The M1 general series may be shown explicitly or implied by their substitution) for $y = 2 + 2x - \frac{7}{2}x^2 + \frac{8}{3}x^3$ oe Must have y = ... and be in ascending powers of A1 Alternative to Q4b for setting $y = 2 + 2x + ax^2 + bx^3$ **M**1 $for(2+2x+ax^{2}+bx^{3})(2a+6bx)+(2+2ax+3bx^{2}...)^{2}+5(2+2x+ax^{2}+bx^{3})=0$ **M**1 for equating constant terms to get $a = -\frac{7}{2}$ A1 for equating coeffs of x^2 to get $b = \frac{8}{2}$ A1 A1 for $y = 2 + 2x - \frac{7}{2}x^2 + \frac{8}{3}x^3$

Question Number	Scheme	Marks
5(a)	I.F. $= e^{\int 2\tan x dx} = e^{2\ln \sec x} = \sec^2 x$	M1A1
	$y \sec^2 x = \int \sec^2 x \sin 2x \mathrm{d}x$	M1
	$y \sec^2 x = \int \frac{2\sin x \cos x}{\cos^2 x} dx = 2\int \tan x dx$	
	$y \sec^2 x = 2\ln \sec x \ (+c)$	M1depA1
	$y = \frac{2\ln\sec x + c}{\sec^2 x}$	A1ft (6)
(b)	$y = 2, \ x = \frac{\pi}{3}$	
	$2 = \frac{2\ln\sec\left(\frac{\pi}{3}\right) + c}{\sec^2\left(\frac{\pi}{3}\right)}$	
	$2 = \frac{2\ln(2) + c}{4}$	
	$c = 8 - 2 \ln 2$	M1A1
	$x = \frac{\pi}{6} y = \frac{2\ln\sec\left(\frac{\pi}{6}\right) + 8 - 2\ln 2}{\sec^2\left(\frac{\pi}{6}\right)}$	
	$y = \frac{2\ln\frac{2}{\sqrt{3}} + 8 - 2\ln 2}{\frac{4}{3}}$	M1
	$y = \frac{3}{4} \left(8 + 2\ln\frac{1}{\sqrt{3}} \right) = 6 + \frac{3}{2}\ln\frac{1}{\sqrt{3}} = 6 - \frac{3}{4}\ln 3$	A1 (4)
		(4) 10 Marks

Question Number	Scheme	Marks
	Alternative: <i>c</i> may not appear explicitly:	
	$y \sec^2 \frac{\pi}{6} - 2 \sec^2 \frac{\pi}{3} = 2 \ln \left(\frac{\sec \frac{\pi}{6}}{\sec \frac{\pi}{3}} \right)$	M1A1
	$\frac{4}{3}y - 8 = 2\ln\frac{1}{\sqrt{3}}$	
	$y = \frac{3}{4} \left(8 + 2\ln\frac{1}{\sqrt{3}} \right) = 6 + \frac{3}{2}\ln\frac{1}{\sqrt{3}} = 6 - \frac{3}{4}\ln 3$	M1A1
NOTES		
Question 5a	L Contraction of the second	
M1 for the $e^{\int 2\tan x dx}$ or $e^{\int \tan x dx}$ and attempting the integration - $e^{(2)\ln \sec x}$ should be seen if final result is not $\sec^2 x$		
A1 for IF	$= \sec^2 x$	
M1 for mu	ltiplying the equation by their IF and attempting to integrate the lhs	
M1dep for attempting the integration of the rhs $\sin 2x = 2\sin x \cos x$ and $\sec x = \frac{1}{\cos x}$ needed. Dependent on the second M mark		
A1cao for a	ll integration correct ie $y \sec^2 x = 2 \ln \sec x (+c)$ constant not needed	
A1ft for re-writing their answer in the form $y =$ Accept any equivalent form but the constant		
must be present. eg $y = \frac{\ln(A \sec^2 x)}{\sec^2 x}$, $y = \cos^2 x \left[\ln(\sec^2 x) + c\right]$		

Notes for Question 5 Continued

Question 5b

- M1 for using the given values y = 2, $x = \frac{\pi}{3}$ in **their** general solution to obtain a value for the constant of integration
- A1 for eg $c = 8 2\ln 2$ or $A = \frac{1}{4}e^8$ (Check the constant is correct for their correct answer for (a)).

Answers to 3 significant figures acceptable here and can include $\cos\frac{\pi}{3}$ or $\sec\frac{\pi}{3}$

M1 for using **their** constant and $x = \frac{\pi}{6}$ in **their** general solution and attempting the simplification to the required form.

A1cao for $y = 6 - \frac{3}{4} \ln 3$ $\left(\frac{3}{4} \text{ or } 0.75\right)$

Alternative to 5b

M1 for finding the difference between $y \sec^2 \frac{\pi}{6}$ and $2 \sec^2 \frac{\pi}{3}$ (or equivalent with their general solution)

A1 for
$$y \sec^2 \frac{\pi}{6} - 2 \sec^2 \frac{\pi}{3} = 2 \ln \left(\frac{\sec \frac{\pi}{6}}{\sec \frac{\pi}{3}} \right)$$

M1 for re-arranging to y = ... and attempting the simplification to the required form

A1cao for
$$y = 6 - \frac{3}{4} \ln 3$$
 $\left(\frac{3}{4} \text{ or } 0.75\right)$

Question Number	Scheme	Marks
6(a)	$z^n + z^{-n} = e^{in\theta} + e^{-in\theta}$	
	$= \cos n\theta + i\sin n\theta + \cos n\theta - i\sin n\theta$	
	$=2\cos n\theta$ *	M1A1
		(2)
(b)	$\left(z+z^{-1}\right)^5=32\cos^5\theta$	B1
	$\left(z+z^{-1}\right)^5 = z^5 + 5z^3 + 10z + 10z^{-1} + 5z^{-3} + z^{-5}$	M1A1
	$32\cos^5\theta = (z^5 + z^{-5}) + 5(z^3 + z^{-3}) + 10(z + z^{-1})$	
	$= 2\cos 5\theta + 10\cos 3\theta + 20\cos \theta$	M1
	$\cos^5 \theta = \frac{1}{16} (\cos 5\theta + 5\cos 3\theta + 10\cos \theta) $ *	A1
		(5)
(c)	$\cos 5\theta + 5\cos 3\theta + 10\cos \theta = -2\cos \theta$	M1
	$16\cos^5\theta = -2\cos\theta$	A1
	$2\cos\theta\left(8\cos^4\theta+1\right)=0$	
	$8\cos^4\theta + 1 = 0$ no solution	B1
	$\cos\theta = 0$	
	$\theta = \frac{\pi}{3\pi} \cdot \frac{3\pi}{3\pi}$	A1
	2 2	(4)
		11 Marks

Question 6a

M1 for using de Moivre's theorem to show that either $z^n = \cos n\theta + i \sin n\theta$ or $z^{-n} = \cos n\theta - i \sin n\theta$

A1 for completing to the given result $z^n + z^n = 2\cos n\theta$ *

Question 6b

- B1 for using the result in (a) to obtain $(z + z^{-1})^5 = 32\cos^5\theta$ Need not be shown explicitly.
- M1 for attempting to expand $(z + z^{-1})^5$ by binomial, Pascal's triangle or multiplying out the brackets. If ${}^{n}C_{r}$ is used do not award marks until changed to numbers
- A1 for a correct expansion $(z + z^{-1})^5 = z^5 + 5z^3 + 10z + 10z^{-1} + 5z^{-3} + z^{-5}$
- M1 for replacing $(z^5 + z^{-5}), (z^3 + z^{-3}), (z + z^{-1})$ with $2\cos 5\theta, 2\cos 3\theta, 2\cos \theta$ and equating their revised expression to their result for $(z + z^{-1})^5 = 32\cos^5 \theta$

A1cso for
$$\cos^5 \theta = \frac{1}{16} (\cos 5\theta + 5\cos 3\theta + 10\cos \theta)$$

Question 6c

- M1 for attempting re-arrange the equation with one side matching the bracket in the result in (b) Question states "hence", so no other method is allowed.
- A1 for using the result in (b) to obtain $16\cos^5\theta = -2\cos\theta$ oe
- B1 for stating that there is no solution for $8\cos^4 \theta + 1 = 0$ oe eg $8\cos^4 \theta + 1 \neq 0$ $8\cos^4 \theta + 1 > 0$ or "ignore" but $\cos \theta = \sqrt[4]{-\frac{1}{8}}$ without comment gets B0
- A1 for $\theta = \frac{\pi}{2}$ and $\frac{3\pi}{2}$ and no more in the range. Must be in radians, can be in decimals (1.57..., 4.71... 3 sf or better)

Question Number	Scheme	Marks
7(a)	$y = \lambda t^2 e^{3t}$	
	$\frac{\mathrm{d}y}{\mathrm{d}t} = 2\lambda t \mathrm{e}^{3t} + 3\lambda t^2 \mathrm{e}^{3t}$	M1A1
	$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} = 2\lambda \mathrm{e}^{3t} + 6\lambda t \mathrm{e}^{3t} + 6\lambda t \mathrm{e}^{3t} + 9\lambda t^2 \mathrm{e}^{3t}$	A1
	$2\lambda e^{3t} + 6\lambda t e^{3t} + 6\lambda t e^{3t} + 9\lambda t^2 e^{3t} - 12\lambda t e^{3t} - 18\lambda t^2 e^{3t} + 9\lambda t^2 e^{3t} = 6e^{3t}$	M1dep
	$\lambda = 3$	A1cso
		(5)
	NB . Candidates who give $\lambda = 3$ without all this working get 5/5 provided no erroneous working is seen.	
(b)	$m^2 - 6m + 9 = 0$	
	$\left(m-3\right)^2=0$	
	C.F. $(y =) (A + Bt)e^{3t}$	M1A1
	G.S. $y = (A + Bt)e^{3t} + 3t^2e^{3t}$	A1ft
		(3)
(c)	$t = 0 y = 5 \implies A = 5$	B1
	$\frac{dy}{dt} = Be^{3t} + 3(A+Bt)e^{3t} + 6te^{3t} + 9t^2e^{3t}$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}t} = 4 \qquad 4 = B + 15$	M1dep
	B = -11	A1
	Solution: $y = (5-11t)e^{3t} + 3t^2e^{3t}$	A1ft
		(5)
		13 Marks

Question 7a

M1 for differentiating $y = \lambda t^2 e^{3t}$ wrt t. Product rule must be used.

A1 for correct differentiation ie
$$\frac{dy}{dt} = 2\lambda t e^{3t} + 3\lambda t^2 e^{3t}$$

A1 for a correct second differential $\frac{d^2 y}{dt^2} = 2\lambda e^{3t} + 6\lambda t e^{3t} + 6\lambda t e^{3t} + 9\lambda t^2 e^{3t}$

M1dep for substituting their differentials in the equation and obtaining a numerical value for λ Dependent on the first M mark.

A1cso for $\lambda = 3$ (no incorrect working seen)

NB. Candidates who give $\lambda = 3$ without all this working get 5/5 provided no erroneous working is seen. Candidates who attempt the differentiation should be marked on that. If they then go straight to $\lambda = 3$ without showing the substitution, give M1A1 if differentiation correct and M1A0 otherwise, as the solution is incorrect. If $\lambda \neq 3$ then the M mark is only available if the substitution is shown.

Question 7b

- M1 for solving the 3 term quadratic auxiliary equation to obtain a value or values for *m* (usual rules for solving a quadratic equation)
- A1 for the CF $(y =) (A+Bt)e^{3t}$
- A1ft for using **their** CF and **their numerical** value of λ in the particular integral to obtain the general solution $y = (A + Bt)e^{3t} + 3t^2e^{3t}$ Must have y = ... and rhs must be a function of *t*.

Question 7c

B1 for deducing that A = 5

M1 for differentiating **their** GS to obtain $\frac{dy}{dt} = \dots$ The product rule must be used.

M1dep for using $\frac{dy}{dt} = 4$ and their value for A in their $\frac{dy}{dt}$ to obtain an equation for B Dependent on the previous M mark (of (c))

A1cao and cso for B = -11

A1ft for using **their** numerical values *A* and *B* in **their** GS from (b) to obtain the particular solution. Must have y = ... and rhs must be a function of *t*.

Question Number	Scheme	Marks
8 (a)	$A = (4\times) \int_0^{\frac{\pi}{4}} \frac{9}{2} \cos 2\theta \mathrm{d}\theta$	M1A1(limits for A mark only)
	$=18\left[\frac{\sin 2\theta}{2}\right]_{0}^{\frac{\pi}{4}}$	M1
	$9\left[\sin\frac{\pi}{2} - 0\right] = 9$	A1
		(4)
(b)	$r = 3\left(\cos 2\theta\right)^{\frac{1}{2}}$	
	$r\sin\theta = 3(\cos 2\theta)^{\frac{1}{2}}\sin\theta$	M1
	$\frac{\mathrm{d}}{\mathrm{d}\theta}(r\sin\theta) = \left\{-3 \times \frac{1}{2}(\cos 2\theta)^{-\frac{1}{2}} \times 2\sin 2\theta\sin\theta + 3(\cos 2\theta)^{\frac{1}{2}}\cos\theta\right\}$	M1depA1
	At max/min: $\frac{-3\sin 2\theta \sin \theta}{\left(\cos 2\theta\right)^{\frac{1}{2}}} + 3\left(\cos 2\theta\right)^{\frac{1}{2}}\cos \theta = 0$	M1
	$\sin 2\theta \sin \theta = \cos 2\theta \cos \theta$	
	$2\sin^2\theta\cos\theta = (1-2\sin^2\theta)\cos\theta$	
	$\cos\theta \left(1 - 4\sin^2\theta\right) = 0$	
	$(\cos\theta=0)$ $\sin^2\theta=\frac{1}{4}$	
	$\sin\theta = \pm \frac{1}{2} \qquad \theta = \pm \frac{\pi}{6}$	M1A1
	$r\sin\frac{\pi}{6} = 3\left(\cos\frac{\pi}{3}\right)^{\frac{1}{2}} \times \frac{1}{2} = \frac{3\sqrt{2}}{4}$	B1
	$\therefore \text{ length } PS = \frac{3\sqrt{2}}{2}, (\text{length } PQ = 6)$	

Question Number		Scheme	Marks	
		Shaded area = $6 \times \frac{3\sqrt{2}}{2} - 9$, = $9\sqrt{2} - 9$ oe	M1,A1 (9) 13 Marks	
NOTE	ES			
Quest	ion 8a			
M1 :	M1 for $A = \frac{1}{2} \int r^2 d\theta = \frac{1}{2} \int \alpha \cos 2\theta d\theta$ with $\alpha = 3$ or 9 4 or 2 and limits not needed for this mark -			
	ignore any shown.			
A1	A1 for $A = (4 \times) \int_{0}^{\frac{\pi}{4}} \frac{9}{2} \cos 2\theta d\theta$ Correct limits $\left(0, \frac{\pi}{4}\right)$ with multiple 4 or $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$ with multiple 2 needed. 4 or 2 may be omitted here, provided it is used later.			
M1 :	M1 for the integration $\cos 2\theta$ to become $\pm \left(\frac{1}{2}\right) \sin 2\theta$ Give M0 for $\pm 2\sin 2\theta$. Limits and 4 or 2 not			
1	needed			
A1cso for using the limits and 4 or 2 as appropriate to obtain 9				
ALTERNATIVES ON FOLLOWING PAGES				

Notes for Question 8 Continued Question 8b for $r\sin\theta = 3(\cos 2\theta)^{\frac{1}{2}}\sin\theta$ or $r^{2}\sin^{2}\theta = 9\cos 2\theta\sin^{2}\theta$ 3 or 9 allowed **M**1 M1dep for differentiating the rhs of the above wrt θ . Product and chain rule must be used. A1 for $\frac{d}{d\theta}(r\sin\theta) = \left\{-3 \times \frac{1}{2}(\cos 2\theta)^{-\frac{1}{2}} \times 2\sin 2\theta \sin\theta + 3(\cos 2\theta)^{\frac{1}{2}}\cos\theta\right\}$ or correct differentiation of $r^2 \sin^2 \theta = 9 \cos 2\theta \sin^2 \theta$ for equating their expression for $\frac{d}{d\theta}(r\sin\theta)$ to 0 **M**1 M1dep for solving the resulting equation to $\sin k\theta = \dots$ or $\cos k\theta = \dots$ including the use of the appropriate trig formulae (must be correct formulae) for $\sin \theta = \frac{1}{2}$ or $\cos \theta = \frac{\sqrt{3}}{2}$ or $\theta = (\pm)\frac{\pi}{6}$ oe ignore extra answers A1 for the length of $\frac{1}{2}PS = \frac{3\sqrt{2}}{4}$ (1.0606...) or of PS May not be shown explicitly. Give this mark if the **B**1 correct area of the rectangle is shown. Length of PQ is not needed for this mark. for attempting the shaded area by their $PS \times 6$ – their answer to (a). There must be evidence of PS M1 being obtained using their θ for $9\sqrt{2}-9$ oe 3.7279....or awrt 3.73 A1 ALTERNATIVES ON FOLLOWING PAGES

Option 1 – using	rsinθwith/without	manipulation of	$\cos 2\theta$ before	differentiation
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Use of $3(\cos 2\theta)^{\frac{1}{2}}\sin \theta$	First M mark
$3(\cos 2\theta)^{\frac{1}{2}}\cos\theta - 3\left(\frac{1}{2}\right)(\cos 2\theta)^{-\frac{1}{2}}(2)\sin 2\theta\sin\theta = 0$ $3(\cos 2\theta)^{\frac{1}{2}}\cos\theta - 3(\cos 2\theta)^{-\frac{1}{2}}\sin 2\theta\sin\theta = 0$	Second (dependent) M mark for differentiating using the product rule A1 awarded here for correct derivative and M1 for setting their derivative equal to 0
$3(\cos 2\theta)^{\frac{1}{2}} - 6(\cos 2\theta)^{-\frac{1}{2}}\sin^2 \theta = 0$ $\cos 2\theta - 2\sin^2 \theta = 0$	Use of $\sin 2\theta = 2\sin\theta\cos\theta$, division by $3\cos\theta$ and multiplication by $(\cos 2\theta)^{\frac{1}{2}}$ simplify the equation but do not provide specific M marks
$(1 - 2\sin^2 \theta) - 2\sin^2 \theta = 0$ $4\sin^2 \theta = 1$	Use of $\cos 2\theta = 1 - 2\sin^2 \theta$ gives next M mark provided a value of $\sin \theta$ or alt is reached with no errors seen
$\sin \theta = \pm \frac{1}{2}$ $\left(\theta = \frac{\pi}{6}\right)$	Value of $\sin \theta$ reached with use of $\cos 2\theta =$ and no method errors seen (arithmetic slips would be condoned) gives final M mark Second accuracy mark given here.

Use of $3(\cos^2\theta - \sin^2\theta)^{\frac{1}{2}}\sin\theta$	First M mark
	Use of $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$ gives 4th
	M mark provided a value of $\sin \theta$ or alt is reached with no errors seen after the differentiation
$3\left(\frac{1}{2}\right)\left(\cos^2\theta - \sin^2\theta\right)^{-\frac{1}{2}}\left(-2\cos\theta\sin\theta - 2\sin\theta\cos\theta\right)\sin\theta$	Second (dependent) M mark for differentiating using the product rule
$+3\left(\cos^2\theta-\sin^2\theta\right)^{\frac{1}{2}}\cos\theta=0$	A1 awarded here for correct derivative and M1 for setting their derivative equal
$-6\left(\cos^2\theta - \sin^2\theta\right)^{\frac{1}{2}}\cos\theta\sin^2\theta + 3\left(\cos^2\theta - \sin^2\theta\right)^{\frac{1}{2}}\cos\theta = 0$	to 0
$-6\sin^2\theta + 3(\cos^2\theta - \sin^2\theta) = 0$	Multiplication by $(\cos^2 \theta - \sin^2 \theta)^{\frac{1}{2}}$,
$4\sin^2\theta = 1$	division by $3\cos\theta$ and use of
	$\cos^2 \theta = 1 - \sin^2 \theta$ simplify the equation but do not provide specific M marks
$\sin\theta = \pm \frac{1}{2}$	Value of $\sin \theta$ reached with use of $\cos 2\theta = \dots$ and no method errors seen
$\left(\theta = \frac{\pi}{1}\right)$	(arithmetic slips would be condoned) gives final M mark
(6)	Second accuracy mark given here.

Use of $3(2\cos^2\theta - 1)^{\frac{1}{2}}\sin\theta$	First M mark
$0.52 \text{ or } 5(2.005 \text{ or } -1)^2 \sin \theta$	Use of $\cos 2\theta = 2\cos^2 \theta - 1$ gives 4th
	M mark provided a value of $\sin \theta$ or
	alt is reached with no errors seen after
	the differentiation
$3\left(\frac{1}{2}\right)\left(2\cos^2\theta - 1\right)^{-\frac{1}{2}}\left(-4\cos\theta\sin\theta\right)\sin\theta + 3\left(2\cos^2\theta - 1\right)^{\frac{1}{2}}\cos\theta = 0$	Second (dep) M mark for differentiating using the product rule
$-6(2\cos^2\theta - 1)^{-\frac{1}{2}}\cos\theta\sin^2\theta + 3(2\cos^2\theta - 1)^{\frac{1}{2}}\cos\theta = 0$	A1 awarded here for correct derivative and M1 for setting their derivative equal to 0
$-6\sin^2\theta + 3(2\cos^2\theta - 1) = 0$	Multiplication by $(\cos^2 \theta - \sin^2 \theta)^{\frac{1}{2}}$,
$4\sin^2\theta - 1$ or $4\cos^2\theta - 3$	division by $3\cos\theta$ and use of
-101 + 203 = 0 - 5	$\sin^2 \theta = 1 - \cos^2 \theta$ or vice versa
	simplify the equation but do not
	provide specific M marks
$1 \qquad \sqrt{3}$	Value of $\sin \theta$ or $\cos \theta$ reached with
$\sin \theta = \pm \frac{1}{2} \text{ or } \cos \theta = \pm \frac{\sqrt{3}}{2}$	Value of $\sin \theta$ or $\cos \theta$ reached with use of $\cos 2\theta = \dots$ and no method
$\sin\theta = \pm \frac{1}{2} \text{ or } \cos\theta = \pm \frac{\sqrt{3}}{2}$	Value of $\sin \theta$ or $\cos \theta$ reached with use of $\cos 2\theta =$ and no method errors seen (arithmetic slips would be
$\sin \theta = \pm \frac{1}{2} \text{ or } \cos \theta = \pm \frac{\sqrt{3}}{2}$	Value of $\sin \theta$ or $\cos \theta$ reached with use of $\cos 2\theta =$ and no method errors seen (arithmetic slips would be condoned) gives final M mark. Second
$\sin \theta = \pm \frac{1}{2} \text{ or } \cos \theta = \pm \frac{\sqrt{3}}{2}$ $\left(\theta = \frac{\pi}{6}\right)$	Value of $\sin \theta$ or $\cos \theta$ reached with use of $\cos 2\theta =$ and no method errors seen (arithmetic slips would be condoned) gives final M mark. Second accuracy mark given here.

Use of $3(1-2\sin^2\theta)^{\frac{1}{2}}\sin\theta$	First M mark
	Use of $\cos 2\theta = 2\cos^2 \theta - 1$ gives 4th
	M mark provided a value of $\sin \theta$ or
	alt is reached with no errors seen after
	the differentiation
$3\left(\frac{1}{2}\right)\left(1-2\sin^2\theta\right)^{-\frac{1}{2}}\left(-4\cos\theta\sin\theta\right)\sin\theta+3\left(1-2\sin^2\theta\right)^{\frac{1}{2}}\cos\theta=0$	Second (dependent) M mark for differentiating using the product rule
$-6(1-2\sin^2\theta)^{-\frac{1}{2}}\cos\theta\sin^2\theta+3(1-2\sin^2\theta)^{\frac{1}{2}}\cos\theta=0$	A1 awarded here for correct derivative
$-6(1-2\sin^2\theta)^2 \cos^2\theta \sin^2\theta + 5(1-2\sin^2\theta)^2 \cos^2\theta = 0$	and M1 for setting their derivative
	equal to 0
$-6\sin^2\theta + 3(1 - 2\cos^2\theta) = 0$	$(2, 2, 3, 3)^{\frac{1}{2}}$
$(5 \text{ m}^{-1})^{-1} (1 - 2 \cos^{-1} \theta)^{-1} = 0$	Multiplication by $(\cos^2 \theta - \sin^2 \theta)^2$,
$4\sin^2\theta = 1$ or $4\cos^2\theta = 3$	division by $3\cos\theta$ and use of
	$\sin^2 \theta = 1 - \cos^2 \theta$ or vice versa
	simplify the equation but do not
	provide specific M marks
1	Value of $\sin \theta$ or $\cos \theta$ reached with
$\sin\theta = \pm \frac{1}{2}$ or $\cos\theta = \pm \frac{\sqrt{3}}{2}$	use of $\cos 2\theta = \dots$ and no method
2 2	errors seen (arithmetic slips would be
(π)	condoned) gives final M mark. Second
$\left(\theta = \frac{\pi}{6} \right)$	A mark given here.

Use of $9\cos 2\theta \sin^2 \theta$	First M mark even if they have a slip on
	the 9 and use 3 but must be $\sin^2 \theta$
$-9(2)\sin 2\theta \sin^2 \theta + 9(2)\cos 2\theta \sin \theta \cos \theta = 0$	Second (dependent) M mark for
	differentiating using the product rule
	A1 awarded here for correct derivative
	and M1 for setting their derivative equal
	to 0
	\mathbf{D}
$-2\sin^2\theta + \cos 2\theta = 0$	Division by $9 \sin 2\theta$ of $18 \sin \theta$ and use of $\sin 2\theta = 2 \sin \theta \cos \theta$, followed by
or $-\sin 2\theta \sin \theta + \cos 2\theta \cos \theta = 0$ leading	$\sin 2\theta = 2 \sin \theta \cos \theta$ followed by
to $-2\sin^2\theta + \cos 2\theta = 0$ or $\cos 3\theta = 1$ (compound angle formula)	equation but not provide specific M marks
	equation but not provide specific in marks
$-2\sin^2\theta + 1 - 2\sin^2\theta = 0$	Use of $\cos 2\theta = 1 - 2\sin^2 \theta$ gives next M
	mark provided a value of $\sin \theta$ or alt is
$4\sin^2\theta = 1$	reached with no errors seen
	Value of $\sin \theta$ or alt reached with use of
$\sin\theta = \pm \frac{\pi}{2}$ or $3\theta = 2\pi$ (from $\cos 3\theta = 1$)	$\cos 2\theta = \dots$ and no method errors seen
	(arithmetic slips would be condoned)
$\left(\theta = \frac{\pi}{2} \right)$	gives final M mark
$\left(\begin{array}{c} 0 & 0 \end{array} \right)$	Second conversely mark siver here
	Second accuracy mark given here.

Option 2 – using $r^2 \sin^2 \theta$ with/without manipulation of $\cos 2\theta$ before differentiation

Use of $9(\cos^2\theta - \sin^2\theta)\sin^2\theta$	First M mark even if they have a slip on the 9
	and use 3 but must be $\sin^2 \theta$
Could be expanded out to $9\cos^2\theta\sin^2\theta - 9\sin^4\theta$ before differentiation in which case the derivative is immediately given by $-18\cos\theta\sin^3\theta + 18\cos^3\theta\sin\theta - 36\sin^3\theta\cos\theta$	Use of $\cos 2\theta = 2\cos^2 \theta - 1$ gives 4th M mark provided a value of $\sin \theta$ or alt is reached with no errors seen after the differentiation
	Second (dependent) M mark for differentiating using the product rule
$9(-2\cos\theta\sin\theta - 2\sin\theta\cos\theta)\sin^2\theta + 9(\cos^2\theta - \sin^2\theta)2\sin\theta$	A1 awarded here for correct derivative and M1 for setting their derivative equal to 0
$-36\sin^3\theta\cos\theta+18(\cos^2\theta-\sin^2\theta)\sin\theta\cos\theta=0$	
$-36\cos\theta\sin^3\theta+18\cos^3\theta\sin\theta-18\sin^3\theta\cos\theta=0$	
$18\cos^3\theta\sin\theta - 54\sin^3\theta\cos\theta = 0$	Division by $18\cos\theta\sin\theta$ and use of
$\cos^2\theta - 3\sin^2\theta = 0$	$\sin^2 \theta = 1 - \cos^2 \theta$ or vice versa will simplify the equation but not provide specific M marks
$1 - 4\sin^2 \theta = 0 \text{ or } 4\cos^2 \theta - 3 = 0$	
$\sin\theta = \pm \frac{1}{2} \text{ or } \cos\theta = \pm \frac{\sqrt{3}}{2}$	Value of $\sin \theta$ or $\cos \theta$ reached with use of $\cos 2\theta =$ and no method errors seen (arithmetic slips would be condoned) gives final M mark
$\left(\theta = \frac{\pi}{6}\right)$	Second accuracy mark given here.

Use of $9(2\cos^2\theta - 1)\sin^2\theta$	First M mark even if they have a slip on the 9
	and use 3 but must be $\sin^2 \theta$
Could be expanded out to $18\cos^2\theta\sin^2\theta - 9\sin^2\theta$ before	
differentiation in which case the derivative is immediately given	Use of $\cos 2\theta = 2\cos^2 \theta - 1$ gives 4th M
by	mark provided a value of $\sin \theta$ or alt is
	reached with no errors seen after the
$-36\cos\theta\sin^{3}\theta + 36\cos^{3}\theta\sin\theta - 18\sin\theta\cos\theta$	differentiation
	Second (dependent) M mark for
	differentiating using the product rule
$9(-4\cos\theta\sin\theta)\sin^2\theta + 9(2\cos^2\theta - 1)2\sin\theta\cos\theta = 0$	A 1 awarded here for correct derivative and
$2(1)^{3}$ $(1)^{3}$ (1)	M1 for setting their derivative equal to 0
$-30\sin\theta\cos\theta + 30\cos\theta\sin\theta - 18\sin\theta\cos\theta = 0$	8
$-2\sin^2\theta + 2\cos^2\theta - 1 = 0$	Division by $18\cos\theta\sin\theta$ and use of
	$\sin^2 \theta = 1 - \cos^2 \theta$ or vice versa will
$2\cos 2\theta = 1$ or $1 - 4\sin^2 \theta = 0$ or $4\cos^2 \theta - 3 = 0$	simplify the equation but not provide specific
	M marks
	It is also possible to use
	$\cos^2 \theta - \sin^2 \theta = \cos 2\theta$ here
$\frac{1}{\sqrt{3}}$ $\frac{1}{\sqrt{3}}$ $\frac{1}{\sqrt{3}}$ $\frac{1}{\sqrt{3}}$	Value of $\sin \theta$ or alt reached with use of
$\sin\theta = \pm - \text{ or } \cos\theta = \pm - \frac{1}{2} \text{ or } \cos 2\theta = -\frac{1}{2}$	$\cos 2\theta = \dots$ and no method errors seen
	(arithmetic slips would be condoned) gives
$\left(\theta = \frac{\pi}{2} \right)$	final M mark
(6)	Second accuracy mark given here.

Use of $9(1-2\sin^2\theta)\sin^2\theta$	First M mark even if they have a
	slip on the 9 and use 3 but must
Could be expanded out to $9\sin^2 \theta - 18\sin^4 \theta$ before differentiation in which	be $\sin^2 \theta$
case the derivative is immediately given by	Use of
$18\sin\theta\cos\theta - 72\cos\theta\sin^3\theta$	$\cos 2\theta = 2\cos^2 \theta - 1$ gives 4th
	M mark provided a value of
	$\sin \theta$ or alt is reached with no
	errors seen after the
	differentiation
	Second (dependent) M mark for
	differentiating using the product
$9(-4\cos\theta\sin\theta)\sin^2\theta + 9(1-2\sin^2\theta)2\sin\theta\cos\theta = 0$	rule
$-36\sin^3\theta\cos\theta - 36\sin^3\theta\cos\theta + 18\sin\theta\cos\theta = 0$	A1 awarded here for correct
$\frac{1}{2} \frac{1}{2} \frac{1}$	derivative and M1 for setting
	their derivative equal to 0
$1 - 4\sin^2\theta = 0$	Division by $18\cos\theta\sin\theta$ will
	simplify the equation but not
	provide specific M marks
1 $\sqrt{3}$ 1	Value of $\sin \theta$ or alt reached
$\sin\theta = \pm \frac{1}{2}$ or $\cos\theta = \pm \frac{1}{2}$ or $\cos 2\theta = \frac{1}{2}$	with use of $\cos 2\theta = \dots$ and no
	method errors seen (arithmetic
	slips would be condoned) gives
	final M mark
$\left(\theta = \frac{\pi}{c} \right)$	Second accuracy month civer
(6)	bere

Using the factor formulae after differentiating $3(\cos 2\theta)^{\frac{1}{2}}\sin \theta$:

M1 awarded for using $3(\cos 2\theta)^{\frac{1}{2}}\sin \theta$

$$3\left(\frac{1}{2}\right)\left(\cos 2\theta\right)^{-\frac{1}{2}}\left(-2\sin 2\theta\right)\sin\theta+3\left(\cos 2\theta\right)^{\frac{1}{2}}\cos\theta=0$$

M1A1 awarded for correct differentiation using product and chain rule

M1 for setting derivative equal to zero

Multiplication by $(\cos 2\theta)^{\frac{1}{2}}$ and division by 3 gives $\cos 2\theta \cos \theta - \sin 2\theta \sin \theta = 0$

$$\cos 3\theta = 0$$

dM1 mark can now be awarded for using correct trigonometric formulae to reduce the equation to $\cos k\theta = ...$ but the A mark requires $\cos \theta = ...$ or $\theta = \frac{\pi}{6}$

$$3\theta = \frac{\pi}{2}$$
$$\theta = \frac{\pi}{6}$$

The A1 mark can now be awarded

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